Tonal Plexus Microtonal Keyboard

Regent for the Future of Music

A musical instrument is more than a work of art for the artisan and a tool of the trade for the musician; it is a product of evolution in industry and technology, and an embodiment of a collective idea in a culture over a span of time. The Tonal Plexus keyboard is not unlike other instruments, being a mixture of art and science; it differs from other instruments mainly in its interpretation of the collective idea that it embodies. In this article I attempt to clarify that idea by presenting the Tonal Plexus keyboard as more than just a thing in itself, by discussing its relationship to history, its theoretical rationale, its geometric organization, and its playability, to clarify and illuminate its reason for being.

1. The King is dead! Long live the King!

The pipe organ has long been called the King of Instruments.[1] In a more general, but no less majestic sense, all keyboard instruments represent a royal lineage, both worshipped and despised. They are the rulers of musical systems, holding all the keys of state, so to speak. The modern keyboard tends to be equated with twelve tone equal temperament (12ET), the most pervasive tuning in modern Western music; however, it is well known that this tuning was not always the law of the land. Its historic succession began with the rule of pure fifths (ca. 500 B.C.E. – 1500 C.E.), followed by the dynasty of the pure thirds meantones (ca. 1500 – 1750), with unequal temperaments in various mixtures of semi-pure thirds and fifths struggling for power beginning around 1650, losing control over the next two centuries to the controversial and now infamous tyrant: 12ET, rejecting the idea of interval purity altogether in favor of a closed circle of slightly flat fifths.[2]

A brief glance at history will show that Western music and the keyboard that represents it are rooted in what today is known as *microtonality*, or tunings other than 12ET. Judging by spans of time,12ET clearly has no claim to historical supremacy. For well over a thousand years, 12ET was not the tuning of choice; instead, a wide variety of tuning resources were embraced, including up to 31 pitches during the meantone era.[3] 12ET was generally rejected as inferior throughout the history it came to inherit, yet it has enjoyed a more stable reign than its proud predecessors. Being the present monarchy, it deserves special attention from those who would consider staging a coup.

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[1] Mozart, “The organ is to my eyes and ears the king of all instruments.” Incidentally, in the same letter, referring to the organ’s lack of capacity to change its loudness by touch as on a pianoforte, Mozart writes, “That makes no difference”.

[2] Lindley, et al.

[3] Davies; Lindley; Stembridge; et al.

12ET sits rather smugly in its inherited domicile, a structure vaguely reminiscent of a Parthenon with four imposing pillars: music theory, notation, instruments, and performance practice, resting on a foundation of music education, crowned by composition.

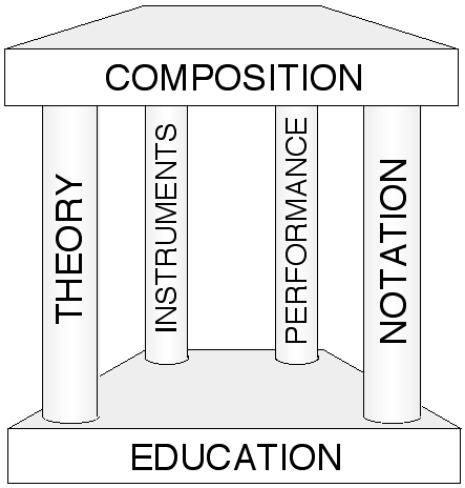

Figure 1

Ironically, each part of the palatial structure exposes its present occupant as a usurper. Western theory, notation, instruments, performance practices, and compositional language evolved as tunings changed, with the music of each era reflecting the tunings of that era. Hundreds of years of historical Western musical practice quite simply had nothing to do with 12ET, yet that tuning now rules the land in luxurious autocratic style, marked by typical propaganda and mass subjugation. The story of 12ET's ascendancy to power may be familiar to those with a knowledge of tuning history, but it lends important context to the present discussion.

The musical staff owes its structure to diatonic pure fifths, from a time when pure fifths were the law and thirds were avoided because their tuning sounded dissonant.[4] An early musical keyboard at that time is said to have been a system of seven or eight diatonic levers, each key having a one-to-one notational counterpart.[5]

In the sixteenth century, when pure thirds had come into vogue, their serenity permeated the works of an era, and staff accidentals owe their present organization to the highpoint of that era, when the difference between a sharp and a flat was something immediately audible as right or wrong, being again linked one-to-one to keys on the keyboard. If a key was tuned to G♯, it was to be used only for the note G♯ and not for the note A♭, which would require a different tuning. By this time, the keyboard pattern was similar to

[4] Bent, et al.

[5] Meeùs

what we now see on harpsichords and organs, famously shown in the organ keyboard at Halberstadt.[6] Modern Western music theory considers Major Thirds to be consonances as a result of the meantone era, yet 12ET renders them as dissonances. Likewise, modern music theory and notation still make clear distinctions between enharmonic accidentals, but 12ET obliterates these distinctions.

Many keyboards of the seventeenth century had subsemitones (split-keys) so that enharmonic sharps and flats could be played, usually with 14 or 16 keys per octave, and sometimes with as many as 31 keys per octave, every note having a one-to-one corresponding key on the keyboard.[7] The different emotional effects of unique pitches, such as G♯ versus A♭, D♯ versus E♭, and so on, were thereby carefully preserved, offering a variety of musical expression not possible with the more widely used keyboards having only twelve keys per octave. This high level of musical clarity lasted little more than a century, until one-to-one correspondences between pitches and keyboard keys started to be abandoned as *well temperaments* took hold. Twelve keys began taking on double-duty, where a single key could for example function for both G♯ and A♭, approaching the modern idea of *enharmonics*, where substantively different pitches collapse into a single practical pitch. Here we reach the most important turning point in the evolution of pitch in Western music history, eventually leading to the ascent of 12ET.

It is unclear when 12ET became the norm for keyboard instruments, but a paradigm shift seems to have taken place in the nineteenth century, by the time the piano had become well established.[8] It should be noted that strictly tuned 12ET is not actually found in the real world on acoustic instruments, but this has not diminished its impact as a ruling paradigm. While instruments of variable pitch – voices, trombones, and strings – have clearly never been limited by any fixed tuning, they are still affected directly by the tuning of keyboard instruments in ensemble situations, and indirectly by ubiquitous keyboard instruments in the general musical environment. All instruments at one time or another must match intonation with a piano. Regardless of whether or not an instrument intones strictly in 12ET, continual exposure to any single tuning tends to reinforce a perception of correctness of that tuning, through simple repetition and unconscious pitch memorization.

In summary, the performance practices of previous eras were keenly attuned to expressive microtonal distinctions of pitch in terms of tuning, spelling, and notation, and the keyboard instruments of those eras were built to produce specific one-to-one pitch relationships in order to preserve the uniqueness of those pitch relationships. Educated musicians of previous eras knew the musical facts concerning expressive differences between pitches such as G♯ and A♭, and those relationships were fiercely upheld. In modern times this has become an antiquated pursuit. What were once considered serious matters of the most tremendous musical significance, at the very heart of the expressive power of music, are now largely

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[6] Preatorius, Fig.25

[7] Stembridge, p.53

[8] Jorgensen, p.1

viewed as trivial matters of mere convenience. Thinking about music in terms of 12ET leads logically to such an attitude. Young music students whisper when their music theory instructor is out of earshot, “G♯ and A♭ are the same thing”, and who can really blame them? It is exactly so, according to the present Ruler of the system; there is, after all, now only one key for both G♯ and A♭ on every modern piano.

Being at odds with the majority of Western history, the ascendancy of 12ET appears tragically ironic; however, viewed in the context of more recent history, its adoption appears quite reasonable. Frequently modulating, highly chromatic music had become standard fare in the nineteenth century, which was also a time of innovation in acoustic instrument design; woodwinds received mechanical keys and brass received valves – inventions which allowed something structurally closer to 12ET to be explored in ensemble music. In the twentieth century, devastating effects of two world wars virtually halted experimentation in pitched instrument design, and the simultaneous emergence of blues, jazz, and rock also began to redefine the place of Western art music in popular culture. Enduring many years of tumultuous upheaval, having been the last focal point of technological progress in instrument making over the previous hundred years, 12ET emerged as the new lingua franca of high brow art in the mid twentieth century, and was also adopted as the default standard for new instruments such as electric guitars and electronic keyboards emerging in the popular music industry. As a result, today we find a marketplace dominated by attractive, high quality, mass produced instruments all built to play in 12ET, implicitly perpetuating the idea that 12ET is the best tuning with which to make music. Countless beautiful, high quality musical instruments await you – never mind that they are all built exclusively for 12ET; why would you want anything else? This situation I refer to as the *musical-industrial complex*.

In the twenty-first century, serious Western art music barely survives among museum offerings of tradition, on the periphery of a consumer culture dominated by ephemeral, disposable products of mass production. In such a tenuous world, the musical-industrial complex gives 12ET protection by authority with the implicit promise of permanence. The pillars of tradition have come to serve as a backdrop for shooting stars, as each new musical work follows a predictable trajectory not unlike that of a firework: rising to premiere, exploding in a moment of glory, and leaving behind the wisp of a contrail quickly obscured by clouds. The architecture of tradition remains unchanged by such bombardment. Some who have dreamed of achieving more than an ephemeral burst have striven to bring about a revolution, a transformation of the accumulated machinery of Western tradition, but dismantling and rebuilding the musical-industrial complex has proven itself to be a dangerous, quixotic, seemingly impossible task.

2. Noble Lessons of the Twentieth Century

Since 12ET ascended to dominate the modern age, no one has been able to transform all the interconnected machinery surrounding it in such a way as to revolutionize the musical-industrial complex, though courageous few have tried. To rise up against the tuning tyrant in our time requires studying the scope of that challenge and the shortcomings of previous challengers. What follows is also familiar

territory for those versed in the subject of tuning, but it deserves special scrutiny in the context of twenty-first century novel keyboard design.

12ET had attained such a stranglehold on twentieth century musical culture that the predominant approach to microtonality at that time consisted of breaking a 12ET whole tone into smaller equal parts. Considering the various tunings possible with that logic, the simplest option, 24 ET (quartertones) was the clear winner, quickly becoming the most widely used non-12ET tuning of the century.[9] Even so, those who championed this approach are still virtually unknown, being overshadowed by well known composers who, by comparison, are mere dabblers in matters of tuning. The real heroes of 24ET are Alois Hába, Ivan Vyschnegradsky, and Julián Carillo, who each independently had keyboard instruments built to play in 24ET, developed unique methods of notation for it, and wrote a considerable number of works in that tuning.[10] Although they each advocated higher divisions of the 12ET whole tone (36ET, 72ET, and 96ET, respectively), and had some keyboards built to play in higher divisions, their compositional outputs tended to favor 24ET.[11] Previously, this tuning had not been widely advocated, because it does not improve 12ET beyond admitting the eleventh harmonic by accident; however, because of its prevalence (it even explicitly made its way into jazz)[12], 24ET became virtually synonymous with microtonality in the twentieth century. Today we can see the damage this has caused. Because music in 24ET tends to sound out of tune, it has created an expectation for all non-12ET tunings to sound out of tune, in turn perpetuating the myth that 12ET sounds more in tune than any other tuning. The stigma continues to this day; an educated musician will say, “Microtonal? Oh yes, you mean quartertones; of course, I’m familiar with that," while thinking to himself, “*that crazy out of tune stuff*.”[13]

Adriaan Fokker rediscovered a different kind of equal division which is superior to 12ET in many respects, Huygens’s 31ET, a meantone-related tuning which can be said to belong to that previous era. Fokker’s rigorous scientific background led him to design a thorough system, including a keyboard, theoretical nomenclature, and notation for 31 tones; however, he was hindered somewhat by a lack of professional connection with the wider musical establishment, though he did influence Hába and Vyschnegradsky. Even having a pipe organ (the King of his system) and electronic instruments built did not allow his work to substantially affect the musical-industrial complex, and his work unfortunately remains little known,

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[9] The historically all-important *tone*, being either a Pythagorean 8:9 or a meantone step between 8:9 and 9:10, being broken into nine commas, and so on, had become so misconstrued in the twentieth century as to be viewed as nothing other than the modern 12ET whole tone.

[10] Benjamin; Criton; Vyslouzíl

[11] ibid.

[12] Ellis

[13] Of course, how 24ET sounds depends upon how it is used to make actual music, and whether it sounds in tune is a matter of subjective perception; I am simply making a point about the cultural perception of this tuning, based on my experiences working with students and colleagues.

though the community supporting its continuance is remarkably vibrant, and new works appear fairly often in this tuning.

Harry Partch believed all equal divisions to be dead ends, and took an adversarial posture towards all established norms, focusing instead on pure 11-Limit Just Intonation, creating his own instruments. Although keyboard instruments are among these, he preferred the traditional keyboard action over novel experiments such as his early Ptolemy keyboard.[14] The majority of Partch's instruments were marimbas and plucked strings. Though seldom noted as such, percussive striking and plucking noises as well as string pitch bending issues may lead listeners to hear Partch's music as sounding wrong and out of tune for reasons having nothing to do with the tuning. An unfortunate fact more often noted is that unless one has Partch's original instruments or replicas thereof, one must attempt to make arrangements of his music for conventional or electronic instruments in order to perform his works. Partch's work is fairly well known, influential to be sure, but widely misunderstood and isolated from the musical-industrial complex.

Preeminent musicologist Martin Vogel advocated strict 7-Limit Just Intonation according to a comprehensive theory and notation system outlined in his *On the Relations of Tone*. Though he mentioned 171ET as an alternative to pure Just Intonation, he never developed a novel keyboard for it;[15] however, keyboards, guitars, and brass instruments were designed to his specifications to produce pure 7-Limit Just Intonation.[16] Only the keyboard (again, a pipe organ, the King of his system) has received much attention from performers, but unfortunately only one such instrument exists.

Many other experimental keyboards were built in the nineteenth and twentieth centuries, most being unique one-offs, not designed as parts of larger systems, in various arrangements of Just Intonation or Equal Temperaments, usually made by amateur musicians, engineers, or scientists, some with accompanying treatises and books. A few modern instruments were built to play in 19ET, another meantone-related tuning, revived in the twentieth century by Joseph Yasser. 53ET was long ago advocated as a desirable system, and R.H.M. Bosanquet had a harmonium built to play it, but only one of those keyboards exists, and no compositions for it have surfaced.[17] 72ET is advocated by many, and some body of work now exists in this tuning, with unrelated schools of 72ET composition existing in Boston and Vienna (using competing systems of notation), but there is no novel 72-tone keyboard instrument.[18]

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[14] Partch, p.219

[15] Vogel, p.350-357, he used a multi-manual Halberstadt to play subsets of 171ET.

[16] ibid. p.373-400

[17] Davies, Tbl.1

[18] Hesse; Sims; et al.

It should also be noted that comparative musicology in the twentieth century emphasized the inadequacy and inappropriateness of the entire Western system – its theory, notation, and keyboard – for nonwestern musics which use pitches other than those of 12ET. This can be seen in the first transcriptions of folksongs by Bartók and Kodály, who both used standard notation with arrows and other markings to attempt to indicate intonation not reproducible on standard keyboard instruments. Likewise, there is widespread false translation of many Asian musics into Western notation, either disregarding actual tunings or using inadequate quartertone approximations.[19] As the spread of Western commercial culture reached virtually every part of the globe, the lingua franca of 12ET, and to a lesser extent its branded microtonal solution 24ET, not only significantly damaged Western perception of musics of the world, but also changed those musics from within, as many Asian cultures now use Western notation to represent their own musics, even though 12ET and 24ET representations are simply false.

A study of history from ancient times to the present thus reveals a collective cultural idea of a non-12ET keyboard, the requirements that such a keyboard should fulfill, and the pitfalls which must be avoided in its creation and promotion. The lessons learned from previous efforts are as follows. First, that the tuning one chooses to advocate can potentially be detrimental to the whole cause of microtonality. Second, that designing a system around a single tuning to the exclusion of all others is a sure path to isolation. Third, that being an outsider to the academic musical establishment, and especially having an adversarial attitude towards that establishment, are considerable hinderances. Fourth, that keyboards which are not tied in with theoretical systems are likely to be forgotten. Lastly, that building a one-off keyboard, or even several keyboards, no matter how wonderful, even if they are tied to a larger system, will have virtually no impact on the musical-industrial complex, being only a few against an army of millions.

3. The Making of an Heir Apparent

With the clear advantage of hindsight, a brief description of an ideal microtonal keyboard is therefore as follows. In order not to damage its cause or to exist in isolation by exchanging one prison for another, its tuning must not only sound better than 12ET, but must be capable of reproducing all the correct tunings of Western music history, including Pythagorean and meantone, and it should also reproduce all nonwestern tunings, becoming the tool of choice for ethnomusicologists and experimental composers. In short, it should do the impossible – *include all possible tunings*. It must also be representative of a complete theoretical system, including consistent nomenclature and notation, where every key on the keyboard has a logical name and a one-to-one notational equivalent, as historical precedent demands. It should be designed in such a way that it reforms and expands the existing order of the Western musical system from within, reclaiming the history that has been long obscured, instead of challenging and overthrowing that order from the outside, separating itself from its own roots. Finally, the keyboard should be produced as numerously and distributed as widely as possible.

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[19] Yarman, p.21f

It seems a tall order. As luck would have it, the seemingly impossible first hurdle of reproducing all tunings has a solution, since within the past century researchers in psychoacoustics have determined that the average threshold of pitch discrimination, or Just Noticeable Difference (JND), which I will denote as simply **J**, is about 1/200 of an octave, or 6 cents.[20]

$$\mathbf{J = 6¢}$$

Not surprisingly, no historical keyboard has sported anywhere near 200 keys per octave,[21] but if all possible pitches are to be accounted for, a number this large is required. The first order of business is thus the handling of this large number of pitches by music theory and notation. Theoretical work must precede the keyboard itself, because a keyboard must serve assuredly as the chief dignitary of a system which is operating discreetly, behind the scenes.

Naming around 200 tones per octave appears rather absurd, until one realizes that not every tone requires a categorical name. There can be fewer nominal categorical *steps* in the system than there are total tones in the system, preferably not exceeding 35 steps, since traditional nomenclature allows for 35 possible nominals (7 letters times 5 accidentals). Each of these categorical steps in turn can have some number of fine-tunings, or *intonations*, again preferably not more than 5, since there are 5 traditional accidentals. Expressed in simple algebra, the large number of tones $\boldsymbol{T}$ should be a product of two smaller numbers, $\boldsymbol{n}$ and $\boldsymbol{m}$, where $\boldsymbol{n}$ is a number of nominal steps of which there are $\boldsymbol{m}$ different intonations per step.

$$\boldsymbol{n \cdot m = T}$$

This approach also makes sense considering that such a large number of steps could be called a continuum, because it reaches the limits of human pitch perception, and within this continuum, there must be *zones* corresponding to perceptible pitch categories; $\boldsymbol{n}$ will represent these zones as equal divisions of an octave. The central question is: how many zones, or categorical pitches, are there in an octave? Here we are reminded of Harry Partch's One Footed Bride, various graphs of intervallic consonance across an octave, and so on.[22] Patrick Ozzard-Low's provisional empirical investigation of interval *zones* is also highly relevant.[23] Ideally, the nominal scale $\boldsymbol{n}$ should correspond exactly to this number of perceptible

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[20] Gelfand; Moore and Glasberg; Wier; Zwicker; Shower and Biddulph; et al.

[21] However, according to Davies, the 5-manual, 104 tone *Reinharmonium* designed by Carl Eitz appears to have passed the halfway mark in 1892.

[22] Partch, p.155; comma-based theories of Sauveur and Telemann also come to mind, are mentioned in following.

[23] Ozzard-Low, Appendix I(B)

categories, or zones. While it would be convenient to have 35 or fewer zones, if the number of zones is greater than 35, then extended accidentals will be needed.

Music theory and notation are so closely linked that investigations into one cannot be undertaken without fully considering the other. The ***n*** times ***m*** approach proves equally useful for both theory and notation. As I have outlined in another article,[24] a linear notation principle can be used for any tuning ***T*** where staff positions correspond to equal scale steps of ***n***, and accidentals written on the staff correspond changes of intonation by equal subdivisions of ***m***. The basic idea is shown below.

nominal steps (n) ***intonations (m)***

Figure 2

The staves of such systems require large numbers of lines, which can be organized using differences in thickness and gray scaling according to basic principles of vision science; for example, a staff of 13 lines is shown below.

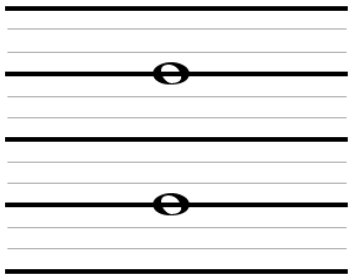

Figure 3

Such a system I call a *complex staff*. The notation of the Perfect Fifth on a complex staff determines the similarity of the system to traditional notation; for example, in the system above, the desired Perfect Fifth spans 12 positions in notation. There is no whole-numbered ET in which the Perfect Fifth spans 12 steps, so this staff does not work for our purposes; however, from this example, it can be seen that systems in which the Perfect Fifth spans a number of scale steps which is divisible by 4 (so that the written interval spans a line to a line with a line in between) will be the most similar to traditional notation. By applying the principle of linear notation, we add another constraint to the solution of $\boldsymbol{n \cdot m = T}$, that the number of steps spanned by the Perfect Fifth of ***n*** should be divisible by 4.

To summarize, the system requires a theoretically ‘good’ tuning with around 200 tones, which can be expressed as ***n*** times ***m***, where ***n*** is also a ‘good’ ET defining perceptible interval zones and having a

[24] Hunt, *A Theory of Musical Staves* (2001)

Perfect Fifth which spans a number of scale steps which is divisible by 4, not requiring ***m*** to be greater than 5.

With so many requirements, it may seem highly unlikely that a solution can be found, but of course there is a solution. The determination of the number of zones, and therefore the zone size, can be solved theoretically by employing the most uncelebrated interval in Western music theory: the *comma*, which I will denote as **Ç**. This is especially appropriate because pitches related by shifts of a comma historically evolved into present day enharmonic pitch categories, or nominals; so this unit should by definition provide a boundary within which changes of intonation do not change the nominal category. The octave has been divided equally into a set of commas by several important theorists and composers, most notably Joseph Sauveur: 43ET, and Telemann: 55ET.[25] Just as an average JND has been established as a perceptual unit **J**, so an average comma can be established as a theoretical unit **Ç**. I found no prior literature on the subject of *average comma size*, so I conducted my own investigations, measuring comma sizes up to the 89th harmonic, ultimately finding an average comma size of 27.4 cents.[26]

Ç = 27.4¢

The ET having a step size closest to 27.4 cents is 43ET at 27.9 cents[27]; however, 43ET has a noticeably flat (meantone) fifth, so it is not the best choice for ***n***. Viewing 43ET as derived from a cycle of Perfect Fifths solves the problem, as collapsing the two smallest intervals of that chain and making the remaining intervals equal in size results in 41ET, which has an almost exactly harmonic Perfect Fifth, which also happens to span 24 steps (which is also divisible by 4).[28] Forty-one also happens to be the provisional number of interval zones suggested experimentally by Patrick Ozzard-Low.[29]

Having found a good value for ***n***, we only need to find the value for ***m***, preferably 5. This turns out to be very simple, because 41 times 5 is 205, which uncannily hits our target, splitting each **Ç** into 5 **J**. The steps of 205ET are actually 5.9 cents, slightly smaller than the average JND, giving *less noticeable error*, or better tuning precision, and the step size of 41ET is slightly larger than the average comma, giving *more distinct scale steps*, or better categorical pitch. 205 also happens to be 1 greater than **12 · 17 = 204**, so it coincidentally contains a scale which is virtually indistinguishable from 12ET.

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[25] Lindley, *Temperaments, 3. Regular mean-tone temperaments to 1600.*

[26] <http://musictheory.zentral.zone/findpage?s=average+comma>

[27] Saveur used this 43ET interval, which he called it the *méride.* It is almost exactly the size of the historical Comma of Archytas, the Septimal comma of 27.3¢.

[28] Fokker (1967) refers to the 41ET division as "*the Von Jankó supracomma."*

[29] Ozzard-Low, Appendix I(B)

Taking our previously noted staff of 13 lines and splitting each space in half results in a staff of 25 lines, where the Perfect Fifth spans 24 steps, which is exactly what is needed for 41ET, where the Perfect Fifth indeed spans exactly 24 steps. As an added bonus, because **7 · 6 = 42** is just 1 greater than 41, distances on this staff are roughly proportional to traditional notation by a factor of 6.

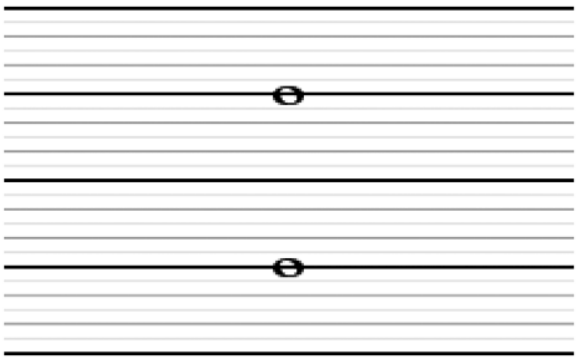

Figure 4

It appears that 205ET is almost a ready-made solution to the whole conundrum; however, 41 nominals is greater than the traditional limit of 35 possible labels (7 letters times 5 accidentals), so the accidentals must be expanded to include triple-sharps and triple-flats, which have in fact been invoked in music theory for centuries, but have never been made standard.[30] The resulting circle of fifths has 41 categories with 49 labels, where the triple accidentals overlap with each other, creating a new set of enharmonics (in the modern sense of being exactly the same pitch). The triple-flat is a straightforward symbol of three flats, while the triple-sharp is a symbol of my design, a variation on the double sharp with a vertical line that forms a shape like an asterisk. The triple accidentals can be seen at the bottom of the circle of 41 fifths shown in Figure 5.

[30] Telemann, p.268

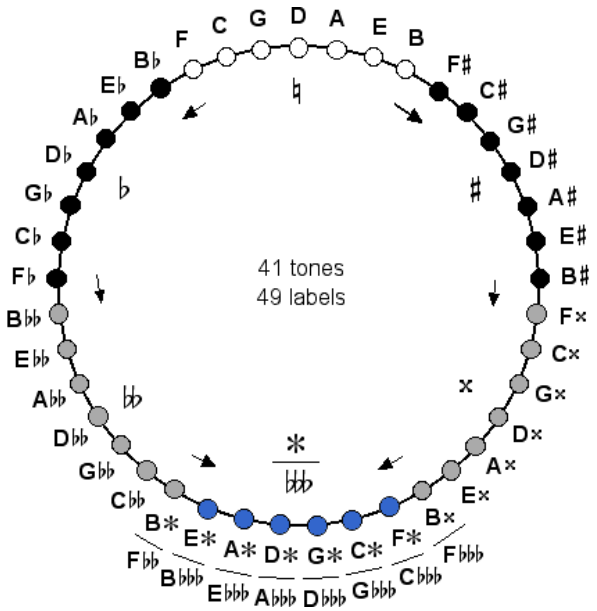

Figure 5

Each of the letter names on the 41-tone circle represents a **Ç** category or zone having five **J** intonations; for example, the note G at the top of the circle can be intoned in five ways, each differing by 1 JND.

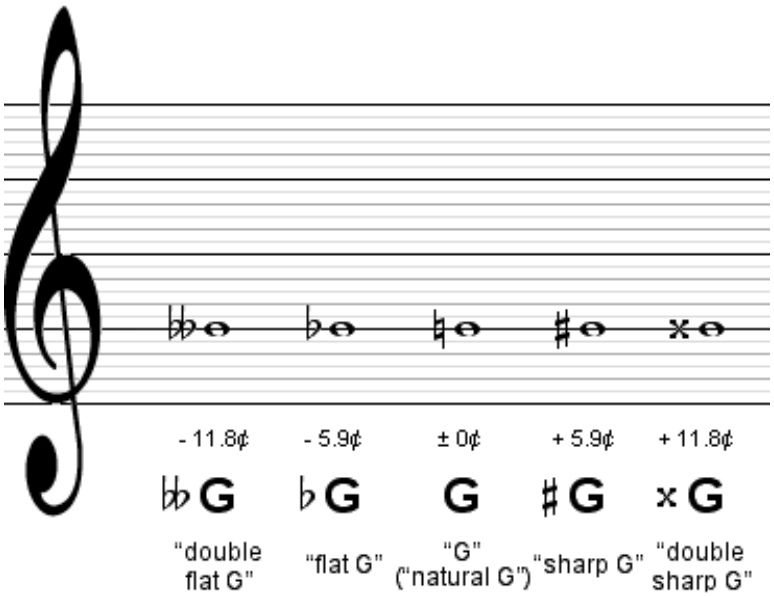

Figure 6

Each of the 41 nominal steps intoned in five ways give a practical realization of the original equation ***n* · *m* = *T***

41Ç · 5J = 205ET

This theory and notation were developed over a period of years before a keyboard was designed to embody the idea as a whole. The system has consistent nomenclature for all pitches and intervals, the full disclosure of which is in my *Systematic Music Theory*.[31]

4. Vanquishing a Lofty Geometry

The 205ET system, being fully developed in both theory and notation, operates behind the scenes while the Tonal Plexus serves as its chief representative; however, before rising up to take up its mantle, the keyboard must first be constructed according to some logical geometry. The present geometry of the Tonal Plexus owes much to the historical Western keyboard, also being influenced by important novel innovations of the past two centuries from Western Europe.

The most important element in the design is the iconic geometry virtually synonymous with Western music, the traditional keyboard as we see it today in modern pianos and organs. The name Halberstadt is sometimes given to this arrangement of keys, referring to the manual of an organ as shown in an early treatise on musical instruments by Praetorius.

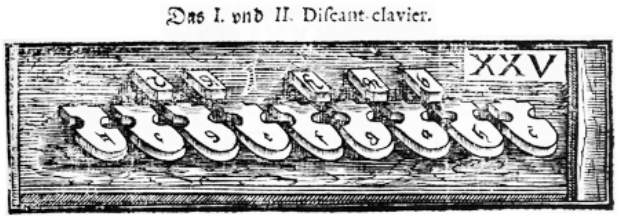

Figure 7 [32]

A challenger to this geometry is found in the accordion family, where keys are rearranged in rows of stacked whole-tone scales, the advantage being that fingering patterns for all scales and chords in any key become the same, a property known as *transpositional invariance*. This treatment was most famously given to the piano by Paul Von Jankó, who patented his design in 1883.[33]

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[31] <http://musictheory.zentral.zone> A summary of the system is included within the help files of Xentone microtonal ear training software, available at <http://hpi.zentral.zone/xentone>.

[32] Preatorius, Fig.25

[33] according to Keislar, p.3. Von Jankó's design is the realization of an idea suggested as early as 1708, by Konrad Henfling.

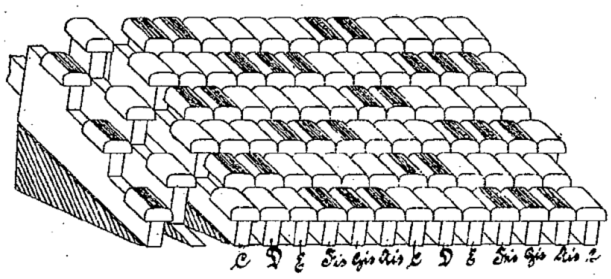

Figure 8 [34]

Although the Von Jankó keyboard provides the advantage of transpositional invariance, it also presents an unfamiliar situation for the modern keyboardist, since it involves increased vertical movement across the keyboard. The manufacturer of such a keyboard is also given new problems because a larger number of duplicating keys is required for the layout to achieve transpositional invariance. For 12-tone keyboards, these problems proved to be a significant hinderance to the acceptance of Von Jankó's design, though many musicians considered the keyboard to be superior to Halberstadt, speculating that it was surely the future of keyboard instruments.[35] Since the nineteenth century, new designs for keyboards having more than 12 keys per octave have usually implemented transpositional invariance, for good reason, since with greater numbers of keys comes greater numbers of possible patterns.

Around the same time as Von Jankó, R.H.M. Bosanquet found that tunings with relatively large numbers of tones based on cycles of Perfect Fifths are logically realized with keyboards resembling stacked zigzag Halberstadt patterns, in arrangements he referred to as "generalised".[36]

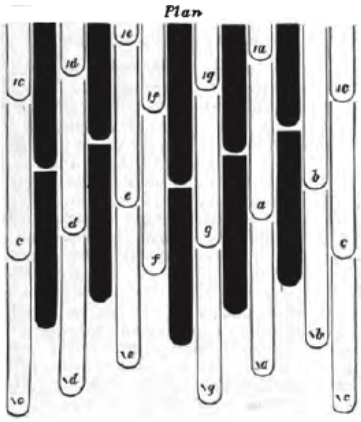

Figure 9[37]

[34] Von Jankó (1885), Fig.7

[35] Dolge, p.79-80

[36] Bosanquet, p.xi

[37] ibid., p.23, Fig.II

In these generalised arrangements, keys rise in pitch from bottom to top, and from left to right, so that a glissando would be played by starting in the leftmost column, playing all the keys in that column starting at the bottom and moving to the top, then moving right one column and doing the same, and so on. This bottom-to-top and left-to-right rising pitch arrangement I will call a *North and East* (N&E) configuration. Such a pattern is transpositionally invariant, as Von Jankó whole-tone patterns are seen within the structure on the diagonal, rather than straight across, yet octaves are found in horizontal straight lines. The Bosanquet pattern can thus be seen as a clever hybrid of Halberstadt and Von Jankó.

Although Bosanquet arrived at his zigzag generalised keyboards by way of tuning arithmetic, such patterns also follow from simple geometric calculation when any large array of keys is arranged in N&E form. Consider the layout of the Fokker organ shown in Figure 10 below. The layout follows N&E arrangement, and it can be clearly seen that the white C key on the right is physically higher than the white C key on the left. In Figure 11, duplicate keys are removed from the picture, and the full octave is repeated to show the overall rising pattern.

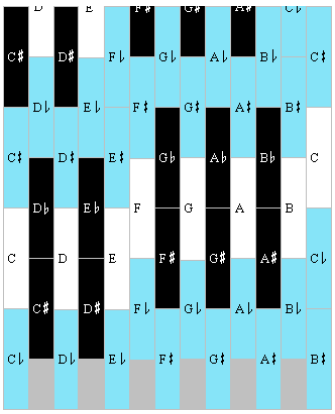

Figure 10[38]

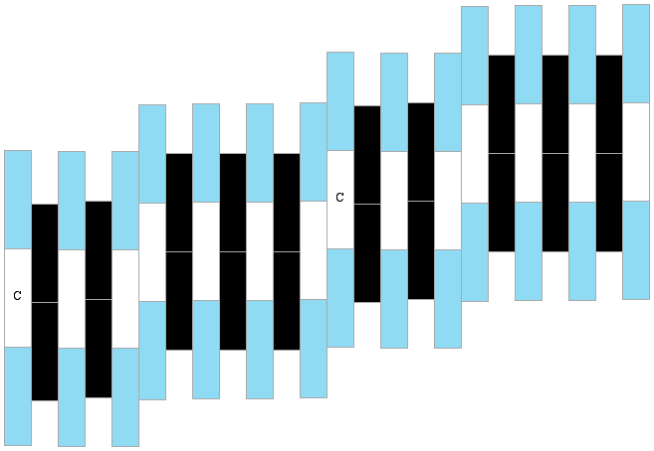

Figure 11

The pattern rises this way from left to right, so that a straight line of keys played from left to right results in a whole tone scale, like the Von Jankó keyboard. The rising of the pattern can be geometrically cancelled by distributing the overall change in height evenly across each column. In geometric terms, this change is simply the slope of the line connecting the C on the left to the C on the right, the ‘rise over the run’. Distribution of this slope results in a pattern which zigzags downwards from left to right, in the manner of Bosanquet’s generalized keyboards, only upside-down.

[38] <http://www.huygens-fokker.org/pics/kb2.png>

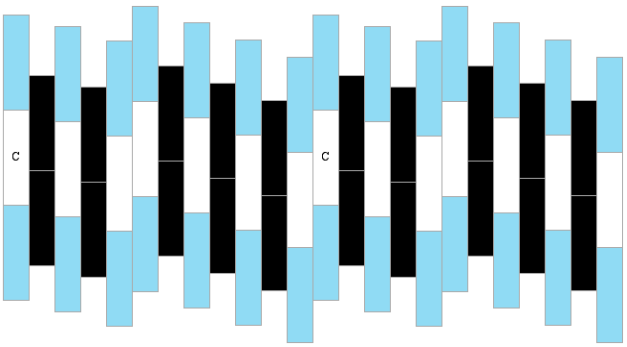

Figure 12

This ‘left-rising’ pattern has to do with the fact that the Fifths of 31ET are flat Meantone Fifths, such that the Thirds are nearly pure. The pattern can just as well be flipped upside down to give a typical Bosanquet right rising pattern, although this implies Perfect Fifths and sharp Thirds. Coloring the Bosanquet pattern Fokker-style makes the similarity of the structures more apparent.

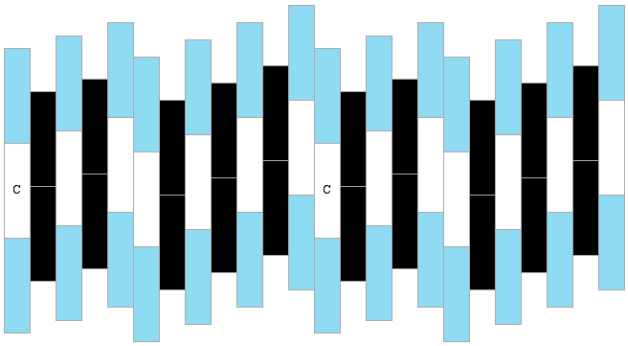

Figure 13

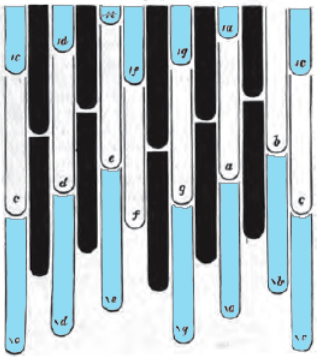

Figure 14

Although I did not consciously work from any keyboard other than the Halberstadt as a model, the keyboards of Bosanquet, Von Jankó (by proxy), and Fokker were all influential to the design of the Tonal Plexus keyboard. Working through numerous designs by trial and error over a number of years eventually led me to favor the transpositional invariance of the N&E arrangement, which in turn led to the present geometry. The resemblances are now fairly obvious, but I only realized after a protracted process that the overall structure of the current Tonal Plexus can be seen as essentially a 41-tone Bosanquet arrangement having a color scheme similar to the Fokker keyboard.

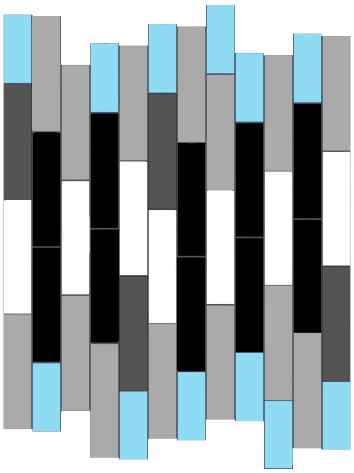

Figure 15

The similarities with Bosanquet should be obvious in the overall zigzag pattern. Similarities with Fokker are perhaps less evident, having to do with color coding. The principle shared by the two keyboards is that each accidental is associated with a color according to its distance away from the center of the circle of Fifths. On both keyboards, naturals are white, while sharps and flats are black. The Fokker keyboard has 'semi-sharps' and 'semi-flats' which are blue, but the traditional circle of fifths has no 'semi' accidentals, having instead conventional double-sharps and double-flats, which are colored gray on the Tonal Plexus. Finally, the triple-sharps and triple-flats which expand the circle of fifths are colored blue. Although it was just said that sharps and flats are black, B♯ and E♯, and C♭ and F♭, are in fact colored dark gray to emphasize the Halberstadt pattern at the center of the keyboard.

A close examination of Figure 15 shows that from left to right each next column is positioned slightly vertically lower than the previous column. This is a unique geometric property of the Tonal Plexus keyboard due to its ***n* · *m* = *T*** theoretical basis. Whereas the right-rising pattern is a vertical offset slope distribution of ***n*** as in a Bosanquet keyboard, this second smaller vertical offset is another distribution of another smaller rising slope resulting from ***m***.

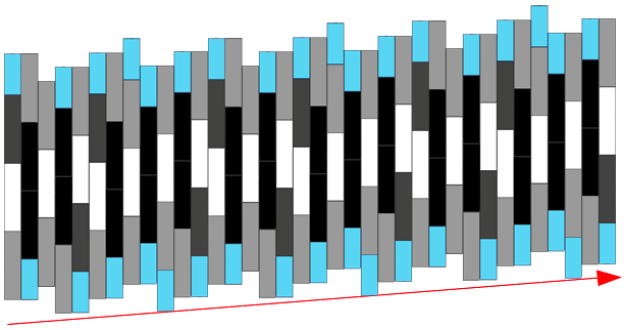

Figure 16

Without correction, the keyboard rises from left to right by the height of one ***m*** per octave. Distributing this small slope allows an octave to span an exactly horizontal line. The 'keys' of the Tonal Plexus are thus revealed as being different than those of a Bosanquet or Fokker keyboard, as the colored areas defining

the overall pattern are not themselves keys, but are rather *regions* of keys, where each region represents an *average* comma **Ç** as a nominal *step* (***n***), and the keys occupying each region represent *average JNDs* **J** as microtonal *intonations* (***m***).

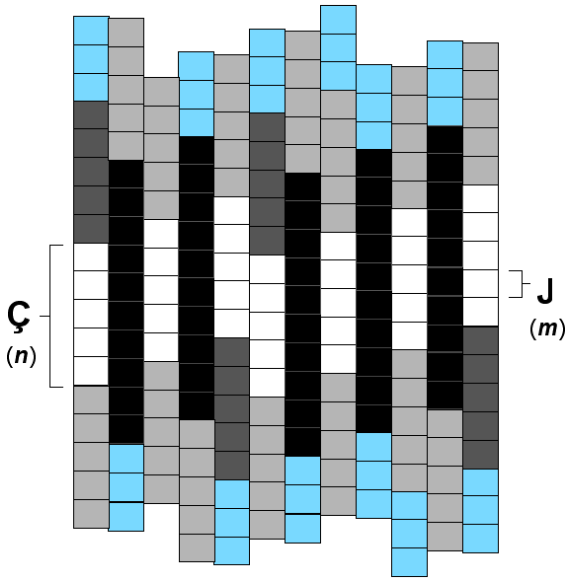

Figure 17

From one column to the next, a downward slope can be seen which is the negative of the slope shown in Figure x, allowing octaves to lie on straight horizontal lines, as shown by the red line connecting three C keys. The negative slope is shown by a yellow arrow below.

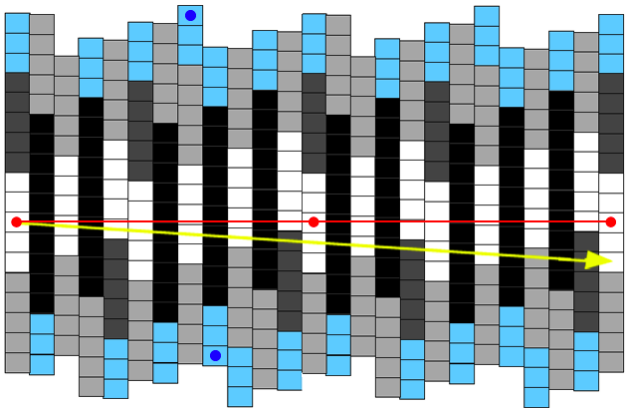

Figure 18

The blue regions at the top and bottom of the structure are smaller than the other regions, having only three keys each, while all other regions have five keys. Each blue region contains one duplicate key; that is, a blue region at the top of one column has its uppermost key repeated as the lowest key in the blue region at the bottom of the next column, as shown above by the two dark blue dots. In this way, each blue region is ‘wrapping around’ at the edges of the pattern, repeating its central key at both the top and

bottom of the keyboard. This is why counting all of the keys in one octave gives a result of 211 rather than 205; there are 6 keys in the enharmonic regions which are duplicated, reflecting the overlapping of the triple accidentals on the Circle of Fifths.

The last detail to be settled is the topology of keys which will ideally allow a player to navigate by touch. The 5 intonations are each given a diameter and elevation according to their distance from the center of each step. The center is the largest key, surrounded on either side by small raised keys, and each region is flanked with keys having a diameter in between these extremes. The pattern corresponds to the notation of 5 intonations per position.

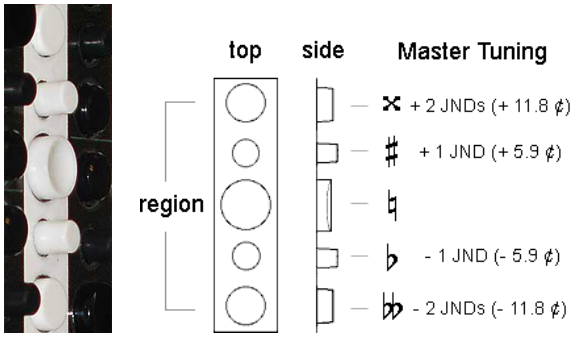

Figure 19

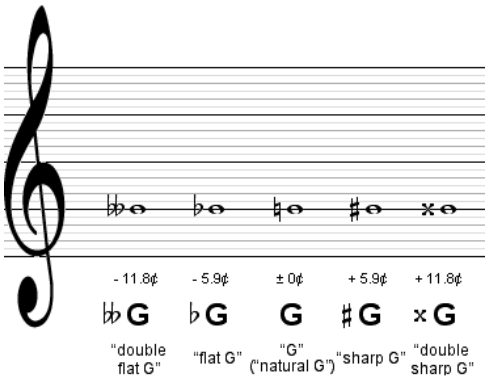

Figure 20

The result is a field of buttons with color, diameter and elevation corresponding to **41Ç · 5J = 205ET**.

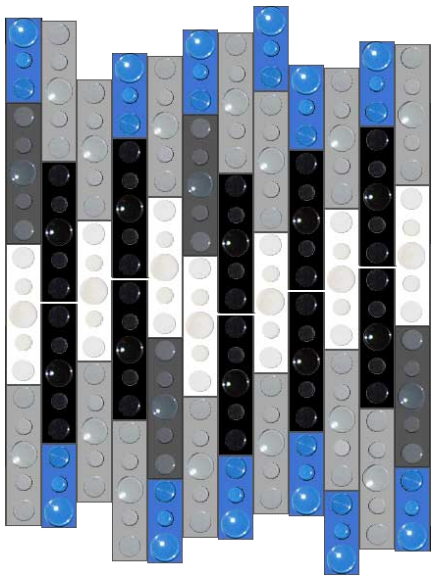

Figure 21

An exploration of how this geometry is used to realize pitches and intervals brings us into the realm of performance practice.

5. Speech From the Throne

The usefulness of a musical instrument lies in its range of expression and its playability. Each member of the instrumentarium enriches the compositional vocabulary idiosyncratically, while keyboard instruments have historically defined the standard lexicon as if by Royal Decree. What then can be expressed with this instrument, and what skill is required to make it speak?[39]

Documentation of all possible combinations of keys would prove very boring, and would not give a useful idea of the range of musical expression of this keyboard; what must be shown is a logical and significant correlation between the structure of the instrument and the basic structures of music. So again we are brought back to the underlying system behind the instrument, to show how the instrument realizes that system, but this alone does not give us what we seek. We are taken back one step further, to define the basic structure upon which the system itself is founded. In essence, we find ourselves back at the beginning, posing a different question than the first, a monumental question, which is: what are the basic structures of music? All musical instruments answer this question in different ways by their very being, through their idiosyncratic structure, as the embodiment of a certain idea of music itself. All fixed pitch instruments to some extent share the idea that discrete, precise and accurate control over pitch is a useful thing for making music. More specifically, a fixed pitch instrument by its nature declares that certain pitches are special. The structure of a keyboard instrument further declares that certain relationships between those special pitches are themselves special. So what is this particular idea of music which is embodied in the Tonal Plexus? What are the special structures at its heart?

There are two answers to this question, corresponding to the ***n · m*** theoretical basis of the system embodied by the keyboard. The first answer has already been given, though not in so many words, declared by ***n***:

> *The basic privileged intervals are those known since earliest Western music history: pure Octaves and Fifths.*

The Circle of Fifths thus serves as the nominal basis of the system, generating the overall geometry which defines the special pitches of the system. The second answer, not yet stated, is declared by ***m***:

> *Subdividing an overall geometry based on Octaves and Fifths reveals all of the intervals discussed for centuries throughout Western music history: the pure intervals of Just Intonation.*

---

[39] Musical nuances other than pitch are not addressed here as they are not specific to the keyboard geometry of the Tonal Plexus; the keyboard can be used to control any sounds.

In the language of Just Intonation, the deep structure is a simplicity of the 3-Limit, and the surface structure is a complexity, practically speaking, without harmonic limit, although the 13-Limit appears within the surface geometry almost as if by magic.[40]

I have already published a full discussion of the 13-Limit harmonic basis for pitches and intervals of the **41Ç · 5J = 205ET** system.[41] For present purposes, it will suffice to say that the fundamental intervals of the system are defined by ***n*** as 3-Limit intervals, and all other intervals derived from higher harmonic limits are considered *comma-shifted* versions of those basic 3-Limit intervals, made available through ***m***. Higher harmonic limits thus create Large, Small, Wide, and Narrow versions of 3-Limit intervals, and all variations of a given interval taken together form an *interval family*, the 3-Limit interval being called the *parent* of the family.

For example, the parent of the Major Second family is the Major Second (M2), a 3-Limit interval, by definition, the ratio 8:9. Two siblings in that family are the Small Major Second (SM2), a 5-Limit interval, 9:10, and the Large Major Second (LM2), a 7-Limit interval, 7:8. These three basic intervals are embedded in the structure of the keyboard by the boundaries of key regions which spell Major Seconds; for example, the C and D regions.

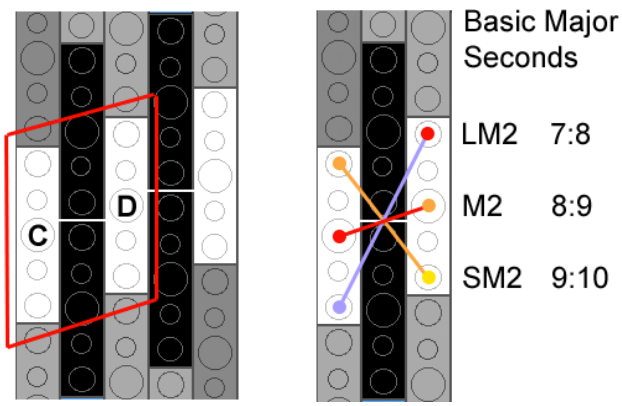

Figure 22

The sibling intervals appear to rotate about an axis at the center of the regions defining the geometry of the parent interval. Figure 21 shows that from C up to D is a Major Second (8:9), from ♯C up to ♭D is a Small Major Second (9:10), and from ♭C up to ♯D is a Large Major Second (7:8). In the same way, Large and Small intervals are found within the geometry of each 3-Limit parent interval.

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[40] I say “as if by magic”, because I did not plan the geometry this way, and did not even realize until some time after designing the keyboard that these patterns are so obvious. A possible third answer to the question also includes the ideas of equal division and unlimited transposition as advantageous – most visibly demonstrated historically by 12ET.

[41] <http://musictheory.zentral.zone>

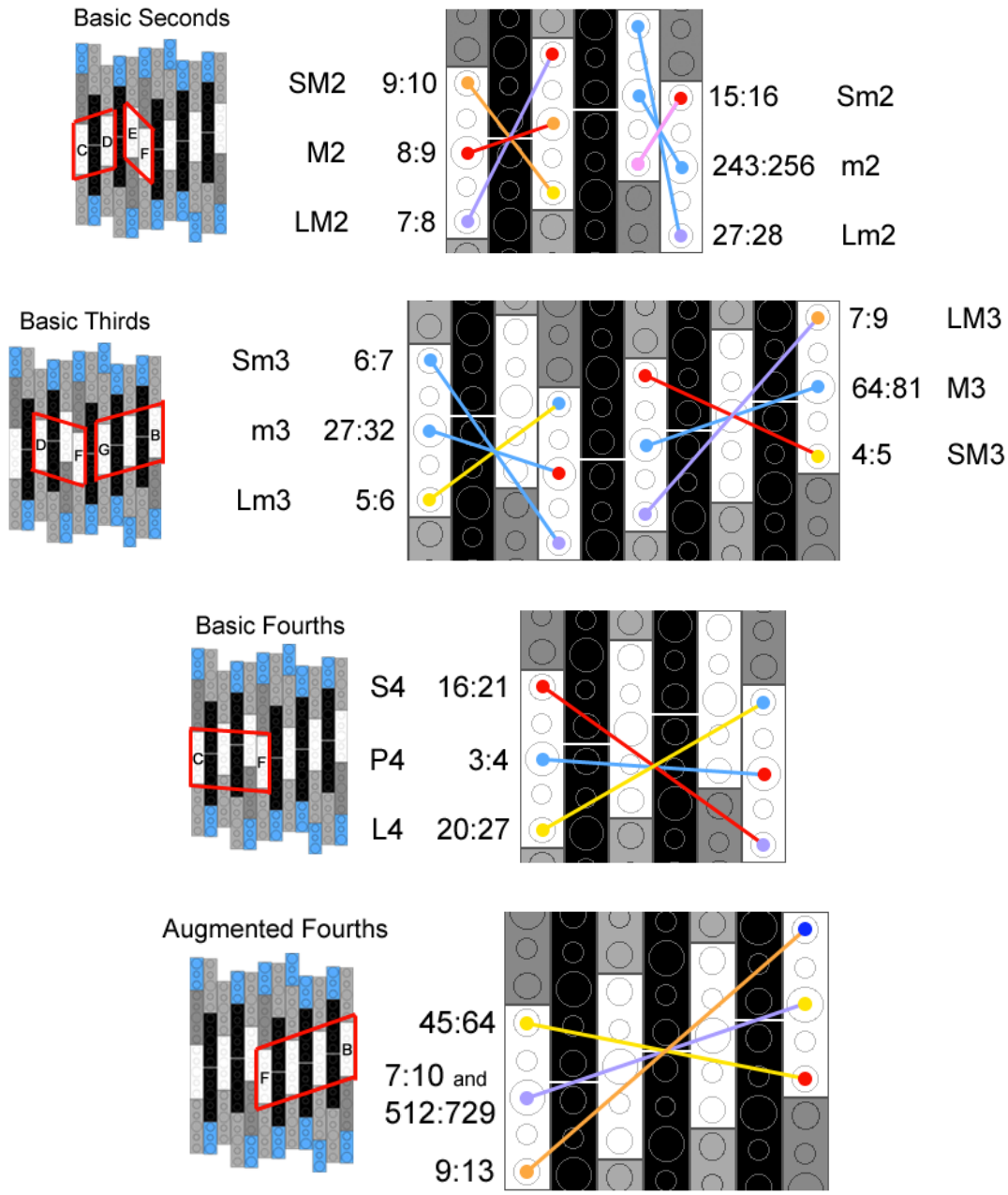
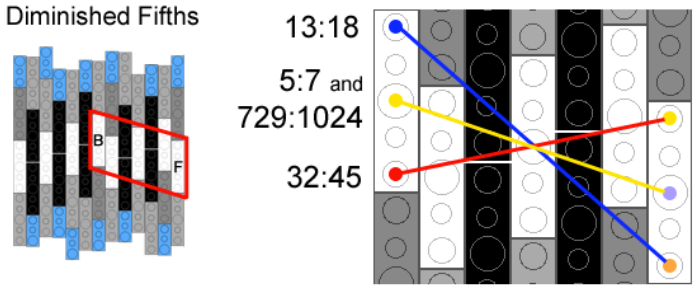


Figure 23

To find families of Fifths, Sixths, and Sevenths, the above structures are simply inverted.



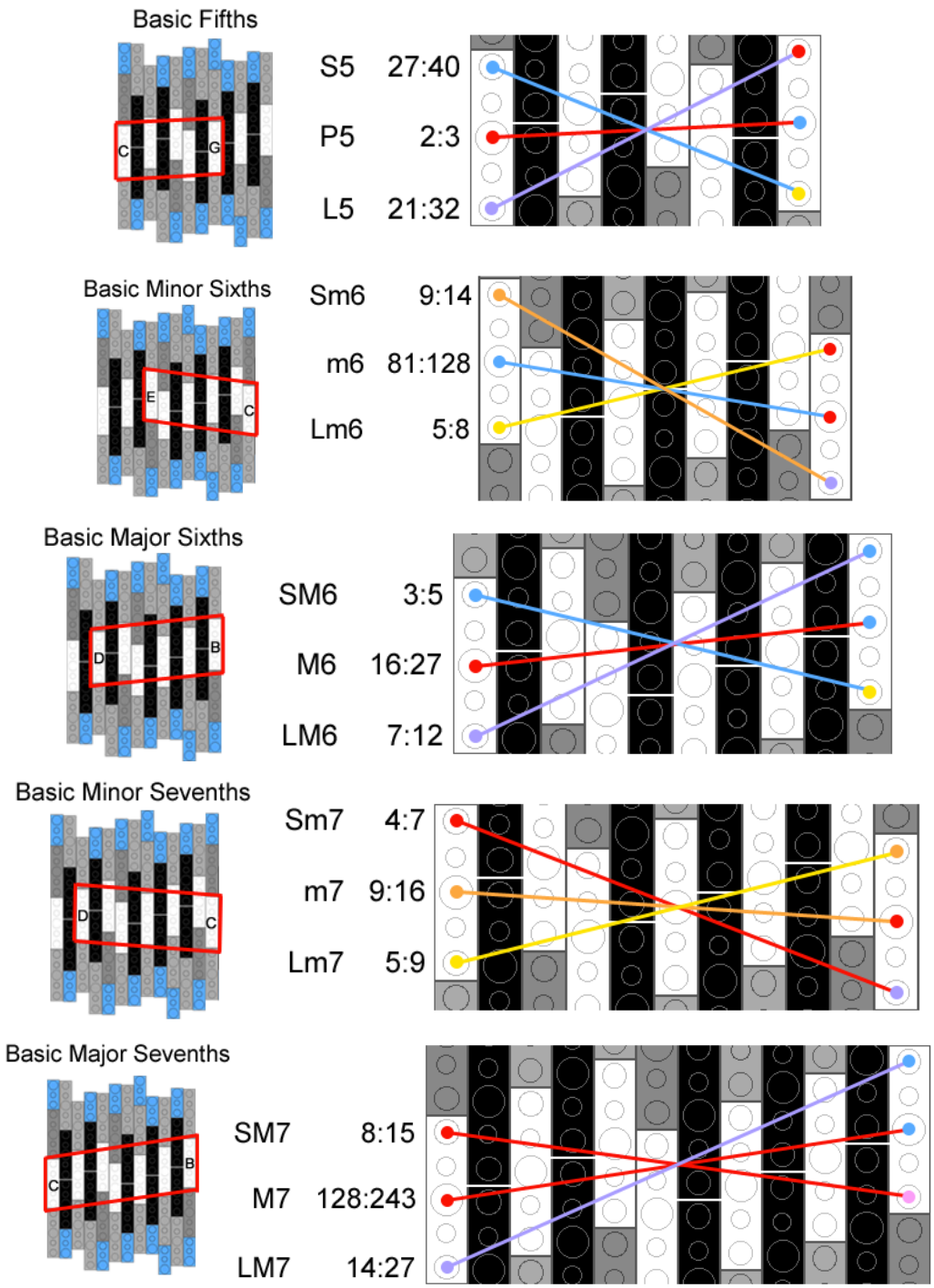


Figure 24

These figures focus on the white regions of the keyboard for purposes of illustration, but the shapes are invariant across the keyboard; they are not restricted to the white keys only. As with any generalised keyboard, the patterns wrap around at the top and bottom edges, so that every interval has exactly two shapes on the keyboard.

Many more Just Intonation intervals are found both in between and outside of those intervals shown in the above figures; for example, additional higher limit Major Seconds exist in between the basic intervals already given.

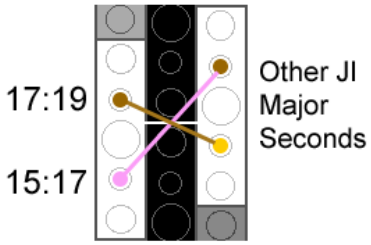


Figure 25

Every combination of keys on the Tonal Plexus can be interpreted as an interval of unlimited Just Intonation with maximum tuning error of less than 3¢. Triads and seventh chords in Just Intonation follow the same clear outlines according to the interval figures shown above.[42] Of course, Just Intonation as a paradigm need not play any role in the conception of music made on a Tonal Plexus. The master tuning is, after all, based on an equal division of the octave; therefore it supports any number of approaches to pitch, and all interval structures are there, with the same level of precision, waiting to be found; the patterns of Just Intonation simply happen to be made apparent by the overall geometry and color coding of the layout, which are themselves generated from an overarching theory that ironically itself had in its inception nothing to do with pure intervals other than the Octave and the Fifth. Such mysterious confluence has represented the mystery of music itself since the beginning of recorded history; the Tonal Plexus thus represents a simple expansion of the traditional system reaching out into unlimited harmonic relationships.

The skill of playing any keyboard instrument lies primarily in playing the right keys at the right time. Beyond that, what makes a performance 'right' is subject to much debate. The technique of playing a Tonal Plexus is somewhat related to pipe organ and harpsichord technique, but of all traditional keyboards, a Tonal Plexus keyboard probably feels closest to the clavichord, because when playing, the fingers tend to have a feeling of delicate connection with the keys. Most pianists have little or no interest in the Tonal Plexus keyboard, because the instrument does not feel like a piano. Though the piano keyboard pattern is obvious, the Tonal Plexus is a very different instrument, after all, with different musical aims than a piano. The most receptive players tend to be people who have not already spent significant time developing conventional piano keyboard technique, and who are excited by the opportunity to learn the technique required by this new instrument, on its own terms.

[42] <http://hpi.zentral.zone/tonalplexus>

6. Ministers of the Crown

It was said that an ideal microtonal keyboard should be the chief representative of a complete theoretical system, including consistent nomenclature and notation, reforming and expanding the existing order of the Western musical system from the inside, reproducing all the correct tunings of Western music history and all nonwestern tunings, essentially *including all possible tunings.* The Tonal Plexus accomplishes all of that.

It is worth noting that no other keyboard has been designed with the goal of accomplishing such aims. The hexagonal grid [43], and the continuous surface [44] are examples of simple geometries which have been developed recently for music. These adaptable surfaces either default to the pitches of 12ET, or define no pitches and intervals as musically special.[45] Novel keyboards based on 12ET represent the status quo, while open-ended designs provide no clear alternative idea. The Tonal Plexus represents something deeper and more musically circumspect, precisely because it was not designed as a thing in itself, but rather as the embodiment of a much larger idea.

In addition to providing a universal default tuning with complete theoretical underpinning, for those who wish to define their own special pitch geometry, all the keys of a Tonal Plexus keyboard can be retuned arbitrarily to any pitch, and the software for this purpose is cross-platform.[46] The software essentially provides a virtual version of the keyboard on screen, showing scale and chord structures, and giving curious musicians a clear idea of what is made possible by the physical keyboard.

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[43] <http://www.starrlabs.com/product/microzoneu648/>

[44] <http://www.hakenaudio.com/Continuum/>, <http://www.apple.com/ipad/> It is debatable whether or not a continuous surface can be called a “keyboard”, since there are no keys, but it is played somewhat in the manner of a keyboard.

[45] Two hexagonal keyboards have been designed to simply rearrange the pitch relationships of 12ET: <http://www.c-thru-music.com/cgi/?page=layout>

[46] <http://hpi.zentral.zone/tpxe>

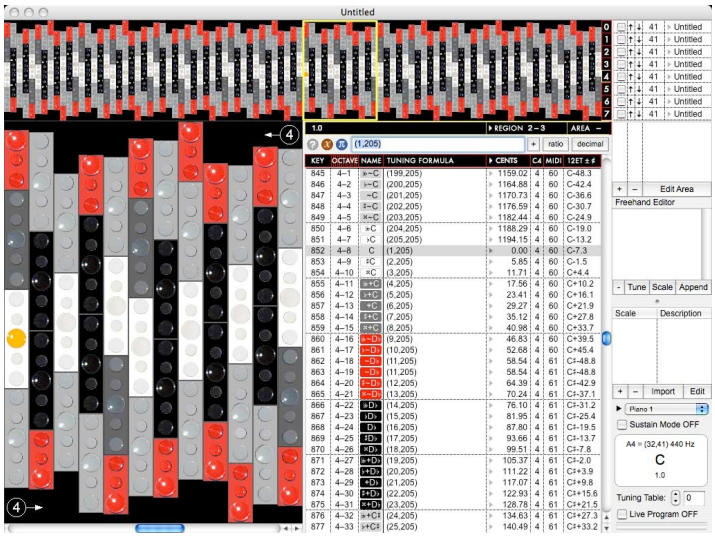

Figure 26

The Tonal Plexus is a young instrument, offering unique and unprecedented musical possibilities to musicians. What is being done with these instruments so far? To my knowledge, the instruments are presently being used for compositional experimentation with tuning, for performance of previously unrealized theoretical ideas, and for demonstrations of tuning issues in early music and avant-garde music. It remains to be seen whether the Tonal Plexus and the system behind it will become the tool of choice for composers, performers, and musicologists, but its potential for attaining that role is clear.

7. A Humble Appeal

The design, production and distribution of a novel keyboard cannot take place without adequate funding. Lacking a fortune and with no corporate backing, between 2006 and 2014, I met the challenges of the marketplace myself, designing several different versions of the keyboard for various needs, including MIDI controllers, MIDI synthesizers, and most recently USB-MIDI devices, building each instrument by hand, and selling them directly to musicians around the world over the Internet. Before I would send a new keyboard on its way to its owner, I often recorded a short video demonstration.[47] As of April 2015, I have stopped building these keyboards, in order to focus on composition. Instead of selling finished keyboards, I have made circuit boards and design files available, so that anyone can build their own Tonal Plexus. The story of the Tonal Plexus is far from over. Many advances and improvements must take place, if and when someone with the means steps forward with necessary funding; I would welcome it.

[47] For example, this experimental performance of J.S. Bach's *C Major Prelude, BWV 846*, in Just Intonation: <http://www.youtube.com/watch?v=72ukYpfpgDI>.

APPENDIX A

A notable historical precedent for my ***n · m = T*** tuning logic is found in the late seventeenth century work of Joseph Sauveur, who used 43 *mérides* per octave, subdividing these units into 7 parts as *eptaméride,* **43 · 7 = 301**. Sauveur used this approach to simplify calculations of interval sizes, since the base 10 logarithm of 2 is 0.301.

It should also be mentioned that two large-numbered ETs historically suggested as “good” tunings are 171ET and 217ET, both of which can be expressed in the form ***n · m = T***.[48] Let us consider each of these tunings.

171ET is 19 times 9, so a system can be constructed by subdividing 19ET into 9 parts; however, the resulting steps are slightly larger than the average JND (7.02 cents), and the nominal fifth of 19ET spans 11 steps (a prime number) so it cannot be notated linearly. 9 intonations per step also requires triple and *quadruple* accidentals to be used for intonations, so that new musical symbols must be invented.

Likewise, 217 can be expressed as 31 times 7; a Fokker keyboard in which each key is broken into 7 parts would represent this system exactly. Unfortunately, the fifth of 31ET spans 19 steps (also a prime), so it can’t be notated linearly, and 7 intonations per step requires *triple* accidentals to be used for intonations.

Considering both tunings, the fifths of both 19ET and 31ET both sound noticeably flat, so that their intonation must be corrected to sound in tune. Because the nominals of traditional Western theory originally come from a diatonic set of pure fifths, it is difficult to justify advocating an expanded system in which those nominals do not correspond to pure fifths; this has always been a problem with meantone systems. 12ET does not have this problem; the one superiority 12ET can claim over any meantone-related equal division is its better sounding fifth.

The reader may investigate other combinations of ***n · m*** having good nominal fifths, such as 53 x 4 = 212, which requires *quadruple* nominal accidentals but whose fifth, spanning another prime number of 31 steps, cannot be divided evenly, so cannot be linearly notated.

APPENDIX B

Below is shown the octave layout of the first prototype Tonal Plexus keyboard. As can be seen by the 2 + 3 black key structure with obvious breaks in the overall pattern, the 12-tone Halberstadt keyboard was the driving principle in this early design. Instead of breaking the octave into 41 regions according to **41 · 5 =**

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[48] Vogel; Secor

205, the overall pattern reflects the equation **12 · 17 = 204 + 1 = 205**, such that keys are grouped into regions of 17 keys each, except for the F♯ region, which has 18 keys. The blue dots show where the 41ET steps fall. Although parts of this layout are transpositionally invariant, between E to F and B to C the continuity is broken.

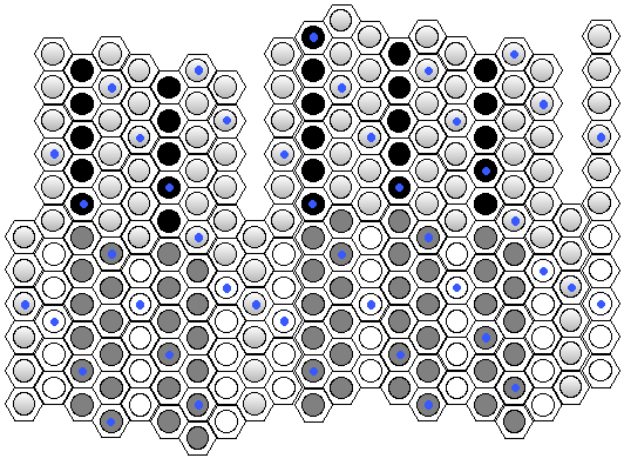

Figure 27

Navigating patterns which vary (that is, which are *not* invariant*)* across a large number of keys is incredibly difficult. After a frustrating year of attempting to teach myself to play this prototype keyboard, I concluded from experience that it was too difficult, and speculated that the generalised approach would be easier. Many subsequent experiments and trials led me to the present geometry, a generalised layout allowing free transposition with remarkable ease. Having spent concentrated effort on both types of designs from the standpoint of performance, I can say without a doubt that the generalised concept is far superior.

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